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NUMERICAL STUDY OF STEADY-STATE REGIMES OF ROTATIONAL—GRAVITATIONAL CONVECTION

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UDC 532.5.013.4:536.252

This article examines the problem of the two-dimensional convection of a viscous incompressible fluid in a rotating horizontal layer with free isothermal boundaries. Approximate solutions are sought by the Galerkin method. We numerically study stability and bifurcative steady-state solutions with a change in the Rayleigh number. The Galerkin method was used in [1-3] to perform calculations for the same problem (also see [4, 5]). In the present investigation, we study transitions in the class of steady-state solutions and calculate the corresponding bifurcative values of R.

Results are presented for a Galerkin system of 62 equations. Equilibria are determined by Newton's method with continuation with respect to the parameter R. We find bifurcative values of R corresponding either to the generation of a pair of equilibria or a shift in the type of stability of the equilibrium. Using the results in [6], we fix the remaining parameters (Prandtl and Taylor numbers, wave number) so that the loss of stability of relative mechanical equilibrium with an increase in R is monotonic. Here, secondary steady-state solutions branch into the subcritical region and are unstable.

Nonetheless, we observed several branches of stable steady motion. These branches appear by different methods with a monotonic increase in R. Of particular interest is the following mechanism: the generation of a pair of unstable equilibria "from air" and their return to stability as a result of Andronov—Hopf bifurcation.

1. Let a viscous heat-conducting fluid fill a horizontal layer of thickness H with nondeformable free boundaries. The temperatures on the lower and upper boundaries of the layer are T_1 and T_2, respectively. In the main regime, the fluid rotates as a rigid body with the angular velocity Ω around the vertical axis. The motion of the fluid is described by the equations of free convection in the Oberbeck—Boussinesq approximation [7, 4]. We will ignore the centrifugal force.

In a cartesian coordinate system (x, y, z) rotating together with the field, the fields of relative velocity $v = (v_1, v_2, v_3)$ and temperature are assumed to be independent of the coordinate y. We introduce the stream function ψ: $v_1 = \partial\psi/\partial z$, $v_3 = -\partial\psi/\partial x$. The equations of motion have the following dimensionless form:

$$\partial\Delta\psi/\partial t = J(\psi, \Delta\psi) + \Delta^2\psi + \tau\partial v/\partial z - G\partial T/\partial x,$$
$$\partial v/\partial t = J(\psi, v) + \Delta v - \partial\psi/\partial z, \ \partial T/\partial t = J(\psi, T) + \mathrm{Pr}^{-1}\Delta T - \partial\psi/\partial x. \quad (1.1)$$

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Here, the chosen units of measurement for length, time t, velocity v, and the deviation of temperature from a linear profile T are H, H^2/ν, ν/H, $T_1 - T_2$; $v = v_2/Re$; $Re = 2\Omega H/\nu$ is the rotational Reynolds number; $\tau = Re^2$ is the Taylor number; $G = g\beta H^3(T_1 - T_2)/\nu^2$ is the Grashof number; $Pr = \nu/\chi$ is the Prandtl number; ν, β, and χ are kinematic viscosity, the coefficient of thermal expansion, and diffusivity; g is acceleration due to gravity; $J(\psi, \theta) = (\partial\psi/\partial x) \times (\partial\theta/\partial z) - (\partial\psi/\partial z)(\partial\theta/\partial x)$.

The following conditions are satisfied at the boundaries of the layer z = 0, 1

$$\psi = \partial^2\psi/\partial z^2 = \partial v/\partial z = T = 0. \tag{1.2}$$

The functions ψ, v, T are assumed to be periodic with respect to x, having the period $L = 2\pi/\alpha_0$. By virtue of (1.2), they can be assumed to be determined over the entire (x, z) plane and periodic with respect to z. Here, the functions have the period 2. Meanwhile,

$$\psi(x, -z, t) = -\psi(x, z, t),\ v(x, -z, t) = v(x, z, t),$$
$$T(x, -z, t) = -T(x, z, t). \tag{1.3}$$

We limit ourselves to solutions which satisfy the additional symmetry conditions

$$\psi(-x, z, t) = -\psi(x, z, t),\ v(-x, z, t) = -v(x, z, t),$$
$$T(-x, z, t) = T(x, z, t), \tag{1.4}$$

and are invariant under the transformation

$$S\colon (x, z) \mapsto (x + L/2, z + 1). \tag{1.5}$$

By virtue of (1.3)-(1.4), periodicity conditions (1.5) are also equivalent to transformation of the central symmetry of the problem relative to the point (x_0, z_0), $x_0 = L/4$, $z_0 = 1/2$.

2. We will apply the Galerkin method to problem (1.1)-(1.5). We seek approximate solutions in the form

$$\psi = \sum_{m,n} \psi_{mn}(t) \exp(i\alpha_0 mx + i\pi nz),$$
$$v = \sum_{m,n} v_{mn}(t) \exp(i\alpha_0 mx + i\pi nz),\ T = \sum_{m,n} T_{mn}(t) \exp(i\alpha_0 mx + i\pi nz), \tag{2.1}$$

where summation is performed over a certain finite set M of pairs of nonnegative integers: $(|m|, |n|) \in M$.

Due to conditions (1.3), (1.4) and the fact that the functions ψ, v, and T are real, the Galerkin coefficients ψ_{mn}, v_{mn}, and T_{mn} are also real. In this case,

$$\psi_{m,-n} = -\psi_{mn},\ v_{m,-n} = v_{mn},\ T_{m,-n} = -T_{mn}; \tag{2.2}$$
$$\psi_{-m,n} = -\psi_{mn},\ v_{-m,n} = -v_{mn},\ T_{-m,n} = T_{mn}. \tag{2.3}$$

It follows from the invariance of the solutions relative to transformation (1.5) that

$$\psi_{mn} = v_{mn} = T_{mn} = 0, \text{ if } m + n \text{ are odd.} \tag{2.4}$$

Thus, Fourier series (2.1) represent expansions of the functions ψ, v, and T in the harmonics $\sin(m\alpha_0 x)\sin(n\pi z)$, $\sin(m\alpha_0 x)\cos(n\pi z)$, $\cos(m\alpha_0 x)\sin(n\pi z)$, respectively, written in complex form.

A system of differential equations for finding $\psi_{mn}(t)$, $v_{mn}(t)$, and $T_{mn}(t)$ was constructed in [8]. It should be noted that it is invariant under the transformation

$$J\colon (\psi_{mn}, v_{mn}, T_{mn}) \mapsto ((-1)^m\psi_m, (-1)^m v_{mn}, (-1)^m T_{mn}), \tag{2.5}$$

corresponding to the shift of the coordinates $x \mapsto x + L/2$. This system is also dissipative if as M we choose any value from the sets (N = 1, 2, 3, ...) $M_N = \{(m, n)\colon\ m + n \le 2N,\ m + n \text{ even}\} \cup \{(2k, 0)\colon\ k \le 2N - 1\} \cup \{(0, 2k)\colon\ k \le 2N - 1\}$.

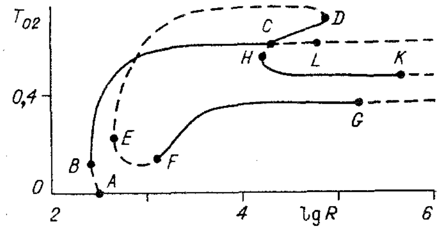

Fig. 1

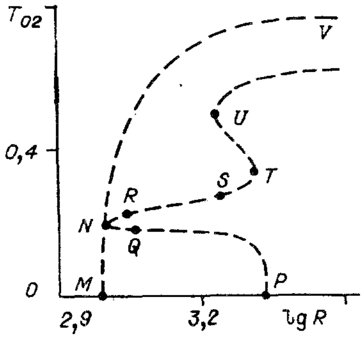

Fig. 2

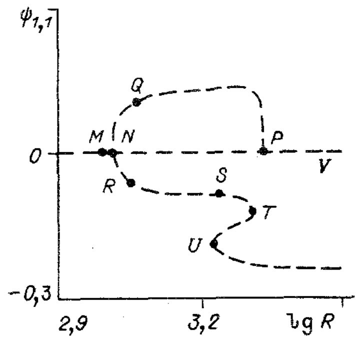

Fig. 3

3. We calculated steady-state solutions for a system of 62 Galerkin equations with $M = M_4$ with allowance for (2.2)-(2.4). It should be noted that, in the case of a nonrotating fluid, this is sufficient for qualitatively accurate reproduction of the bifurcative diagram of the complete Boussinesq equations in the investigated region of parameters [9].

The equilibria of the Galerkin system were calculated by Newton's method with continuation with respect to a certain parameter – the Rayleigh number $R = \Pr \cdot G$. We constructed equilibrium curves in the direct product of the phase space of the system and the axis of values of R. The remaining parameters of the problem were fixed: $\Pr = 0.05$; $\tau = 4$; $\alpha_0 = \pi/4$. Stability of the equilibria was determined from the spectrum of the matrix of the linear system for perturbations. We sought bifurcative values of R corresponding to the generation of a pair of equilibria or a change in the type of stability of equilibrium. The corresponding branch points divide the curve of equilibria into separate sections – branches of equilibria.

The results are shown in Figs. 1-3. The Galerkin coefficients T_{02} and ψ_{11} are plotted off the vertical axis, while the Rayleigh number is plotted off the horizontal axis. The figure was not drawn to scale in order to make the results clearer. Only the mutual location of the equilibrium branches remain the same. We calculated the following bifurcative values of R for all of the points: 667.7 (A), 665.0 (B), 20,480 (C), 73,690 (D), 795.7 (E), 1550 (F), 175,000 (G), 19,680 (H), 450,000 (K), 57,500 (L), 1051.5 (M), 1054.1 (N), 1933.4 (P), 1080 (Q), 1070 (R), 1800 (S), 1939.4 (T), 1580.8 (U).

3.1. The losses of stability of the main regime of motion (rigid-body rotation of the fluid) with an increase in R may be monotonic or oscillatory [4, 7]. At the chosen Pr, τ, and α_0, the loss of stability is monotonic.

For normal perturbations, dependent on (x, z) through the multiplier $\exp(im\alpha_0 x + in\pi z)$, the critical values of R associated with monotonic instability are determined by the equality

$$R_{m,n}(\alpha_0, \tau) = \left[(m^2\alpha_0^2 + n^2\pi^2)^3 + \tau n^2\pi^2\right]/(m\alpha_0)^2. \qquad (3.1)$$

At $\alpha_0 = \pi/4$ (corresponding approximately to the minimum of the critical $R_{3,1}$) and $\tau = 4$, we find from (3.1) that $R_{3,1} = 667.7$; $R_{5,1} = 1051.5$; $R_{1,1} = 1933.4$. If $R < R_{3,1}$, then the main regime of motion is stable. When R, undergoing an increase, passes through the critical $R_{m,n}$, a pair of secondary steady-state solutions branches off from the main solution. They correspond to motions of the fluid which are periodic along x and z with the wave numbers $\alpha = m\alpha_0$ and $\gamma = n\pi$, and they change into one another with the shift $x \mapsto x + \pi/\alpha$. The corresponding equilibria of the Galerkin system belongs to the invariant subspace $P_{m,n}$, in which all of the Galerkin coefficients except $\psi_{km,\ell n}$, $v_{km,\ell n}$, $T_{km,\ell n}$, $v_{2km,0}$, $T_{0,2\ell n}$ are equal to zero (here, k and ℓ are natural numbers). The shift $x \mapsto x + \pi/\alpha$ establishes a transformation of the coordinates $J_{m,n}$ on $P_{m,n}$ which is analogous to (2.5). Meanwhile, $J_{1,1} = J$. The equilibria generated at $R = R_{m,n}$ are converted to one another by the transformation of $J_{m,n}$. We will henceforth use $O_{m,n}$ to denote those equilibria corresponding to a positive value of ψ_{mn}.

At $R = R_{3,1}$ and $R = R_{1,1}$, secondary equilibria branch into the subcritical region. At $R = R_{5,1}$, they branch into the supercritical region; all of them are unstable. Nevertheless, we observed several branches of stable equilibria in our calculations (see below).

In the figures, AB, MN, and PQ represent branches of secondary equilibria $O_{3,1}$, $O_{5,1}$, and $O_{1,1}$. The lines with the points A, B, C, and L (in Fig. 1) and M, N, and V (in Figs. 2 and 3) correspond to equilibria for invariant subspaces $P_{3,1}$ and $P_{5,1}$, respectively. All of the other lines correspond to equilibria for which the phase coordinates are generally nontrivial.

3.2. The generation of stable equilibria is associated with loss of stability of the main regime of motion at $R = R_{3,1}$. The solid line shows the branches of these equilibria in Fig. 1, while the dashed line shows the unstable branches. The letters denote the branch points of the equilibria. The values of R which correspond to them were indicated above. Indicated in Fig. 1 for each branch is the number of characteristic equilibrium numbers lying to the right of the imaginary axis: 1 (AB), 0 (BC), 1 (CL), 0 (CD), 1 (ED), 2 (EF), 0 (FG), 1 (HC), 0 (HK), 1 (MN), 1 (NQ), 3 (PQ), 1 (NR), 3 (RS), 1 (ST), 2 (TU).

At point A, the main regime becomes unstable when $R = R_{3,1}$. Unstable secondary equilibria $O_{3,1}$ branch into the subcritical region (branch AB). With an increase in R, a pair of steady-state solutions (point B) materializes "from air." One of them (branch BC) is stable. These equilibria become unstable at point C as a result of bilateral bifurcation (the simple characteristic number of the equilibrium passes through zero). Here, new equilibria (branch CD) inherit the stability of branch BC; the equilibria of the old branch remain stable in the subspace $P_{3,1}$. The type of stability of the equilibrium changes at point L as a result of bifurcation of the origination of the limit cycle.

The bifurcative generation of a pair of equilibria "from air" is also seen at point H. The equilibria of the branch HK are stable. Their loss of stability at point K is oscillatory and is accompanied by the origination of the limit cycle.

The stable equilibria of branch FG arise with an increase in R as a result of two successive bifurcations. First a pair of unstable equilibria arise at point E "from air." The equilibria of branch EF have two positive characteristic numbers. These equilibria subsequently merge and form a complex-conjugate pair. They return to the left half-plane at point F. The loss of stability by the equilibria at point G also occurs as a result of bifurcative formation of the limit cycle.

3.3. Subsequent bifurcations of the main regime of motion generate only unstable equilibria. The results of calculation of these equilibria are shown in Figs. 2 and 3, where the notation is the same as in Fig. 1. Points M and P correspond to the critical values $R = R_{5,1}$ and $R = R_{1,1}$, while branches MN and PQ correspond to secondary equilibria $O_{5,1}$ and $O_{1,1}$. Of interest is the branch point N, at which the equilibria $O_{5,1}$ have a simple zero characteristic number. At this point, a pair of equilibria not belonging to $P_{5,1}$ branch off from $O_{5,1}$, lying in the invariant half-space $P_{5,1}$. The branch NQ at point Q joins with the branch of secondary equilibria PQ. It should be noted that the equilibria belonging to $P_{5,1}$ are projected on the axis $\psi_{1,1} = 0$ in Fig. 3.

We also observed other equilibrium branch points in the calculations. In particular, the points Q, R, and S correspond to bifurcative formation of the limit cycle, while points T and U correspond to bifurcative generation and disappearance of a pair of equilibria. However, none of these transitions leads to the formation of stable equilibria.

4. Let us form the main conclusions from our study. Within the range of parameters investigated in the phase space of the given system, there are a fairly large number of equilibria with different wave numbers that are multiples of α_0. In particular, four branches of stable equilibria are seen.

The paths by which stable equilibria originate also differ with a monotonic increase in R. They can arise together with unstable equilibria in the bifurcative formation of a pair of equilibria "from air." They can form from an unstable equilibrium as a result of bifurcation of the limit cycle. Finally, they can also occur in the case of bilateral bifurcation, associated with an exchange of stability between two equilibria – one inside the invariant subspace and one outside it.

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USE OF A THREE-COMPONENT MODEL TO COMPUTE GAS SUSPENSION FLOW AND RAREFIED FLOW OVER BODIES

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A 4-component model to describe flow of a suspension (gas with solid particles) over bodies is proposed in [1]. The suspension is a mixture of four components: carrier gas and three kinds of particles, which do not collide with incident s particles, orderly moving reflected r particles, and randomly moving t particles. It is postulated that any two colliding particles (only pair collisions are considered) occur in type t. The particles are assumed to be identical spheres whose diameter d_0 is much less than the characteristic body dimension, while the density ρ_0 is much larger than that of the gas. The velocity distribution of the t particles is assumed to be nearly Maxwellian, and for the t component we use certain results of kinetic theory obtained for a gas consisting of spherical molecules. Here we neglect the influence of resistance of the carrier gas and the possible inelasticity of collisions on the form of the formulas for flux of mass, momentum, and energy. These factors are accounted for in computing the kinetic energy of random motion of particles U_t, determined from the balance equation, which has terms describing dissipation of this energy due to the above causes.

The hypotheses listed, described in detail in [1], lack a rigorous basis, but with them we can construct a rather simple suspension model accounting for random motion of particles, and correctly describing the screening influence of reflected particles, as shown by comparing the computations of [2] with experimental data [3].

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